NUMERICAL SIMULATION OF THREE DIMENSIONAL OSCILLATORY FLOW IN HALF-ZONE BRIDGES OF LOW Pr FLUIDS

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A set of numerical simulations was conducted to understand characteristics of oscillatory Marangoni convection in half-zone liquid bridges with various aspect ratios (from 0.6 to 2.2) and Prandtl numbers (from 0 to 0.02) by a finite difference method. The simulation results indicated that under smaller temperature differences the flow in the liquid bridge is axisymmetric but it becomes unstable against a three dimensional disturbance beyond a certain threshold value of temperature difference. The flow becomes steady three dimensional. This steady flow becomes unstable against time dependent three dimensional disturbances beyond a second critical condition. The numerical simulations revealed the critical conditions, 3-D structure of disturbances and oscillation modes. The first critical conditions showed good agreements with those of linear stability analyses. The second critical conditions also agreed with previous values and gave new critical values for wide range of aspect ratio. Based on these simulations, a flow map was proposed for Pr=0 fluid. Critical Reynolds numbers, flow mode and types of oscillations were also determined for Pr=0.01 and 0.02 fluids.

Keywords: Marangoni flow, Oscillatory flow, Critical Marangoni number, Low Pr fluid, Flow mode

1. INTRODUCTION

Marangoni convection in a half-zone liquid bridge of length *L* and radius *a* confined between two differentially heated isothermals solid disks has become over the years a typical model for the study of Marangoni flows, their stability, and their bifurcations. The stability of free convection in non-isothermal liquid bridges with cylindrical free surface has been the subject of intense research. These studies are stimulated by the experimental fact that flow instabilities in such configurations is responsible for the appearance of striations in crystals grown by floating zone technique under microgravity. It is well known that the flow exhibits axisymmetric and steady toroidal roll cell structure if the temperature difference between the two disks is small and that it becomes unstable and a three-dimensional Marangoni flow arises when the applied temperature gradient exceeds a certain threshold value. On this subject, there have been many experimental works, theoretical studies by means of the linear stability analyses and non-linear numerical simulations. Experiments [1-3] performed with half zone liquid bridges of transparent and high Prandtl number liquids revealed that 3-D Marangoni flow starts always in an oscillatory mode (Hopf bifurcation). Recent developments of computers enabled large scale numerical simulation of the non linear and time-dependent Navier-Stokes equations. So far, several numerical investigations in the case of high Prandtl number liquids have become available [4-8]. Yasushiro et al. [4,5] , Zeng et al. [6] and Lappa et al. [7] analyzed the influence of the aspect ratio on the time-dependent threedimensional structure of Marangoni flow for *Pr*=1, 16 and 30 respectively, elucidating many features of the supercritical flow (e.g. the oscillation type, *standing wave* or *travelling wave*). Shevtsova et al. [8] studied the influence of the temperature-dependent viscosity on the supercritical flow field for $1<Pr<4$. Recently, Tang et. al. [9] reported a transition from axisymmetric steady flow to a steady 3-D flow in a fat half-zone of high Prandtl number fluid.

Linear stability analyses (Neitzel et al.[10], Kuhlmann and Rath [11], Wanschura et al. [12], Chen et al. [13], Chen and Hu [14], Chen et al. [15]) have confirmed that for high Prandtl numbers the instability is oscillatory (Hopf bifurcation) whereas for low Prandtl numbers the instability breaks the spatial axisymmetry (but the flow regime is still steady) prior to the onset of time dependent flow field. Rupp et al. [16], Levenstan and Amberg [17] and Leypoldt et al. [18] found that for low Prandtl number fluids, the first bifurcation is stationary i.e., the supercritical three-dimensional state is steady, and that the flow becomes oscillatory only when the temperature difference is further increased. Lappa and Savino [19] studied the three dimensional structure (the azimuthal wave number of the 3-D disturbance) of the flow pattern after the steady bifurcation for Pr=0.04. Imaishi et al. [20,21] and Yasushiro et al. [22] depicted in detail the complex spatio-temporal behavior of the flow field that occurs after the second (oscillatory) bifurcation of the Marangoni flow for different values of the aspect ratio（defined as ratio of the length and of the radius of the liquid bridge, i.e. $As=L/a$）and of the Prandtl number ($0\leq Pr\leq 0.02$), elucidating different oscillatory behaviors.

Since it is very difficult to conduct well-controlled experiments with small Prandtl number fluids (mostly liquid metals) due to opacity, reactivity and high melting temperatures, there are only few experiments on the flow instability in half zone liquid bridges of semiconductor materials [23,24,25] and molten tin [26]. During on ground experimentation, sounding rocket missions and other parabolic flights, using a X-ray radiography with zirconium-cored tracers, Nakamura et al. [23,24] and Hibiya et al. [25] investigated the structure of the supercritical flow in a half-zone liquid bridge of molten silicon. Their experimental results gave some puzzling aspect and suggested that the oscillatory flow in silicon melt becomes very complex at large temperature differences. Then it is very important to investigate the experimental results by comparing the results of numerical simulations. In this report, we investigated the first and the second critical conditions for small Prandtl number fluids as a function of the aspect ratio. Especially, detailed calculations were conducted for zero Prandtl number fluid. In this case the instability is hydrodynamic in nature i.e., its mechanism does not involve a coupling between the temperature and the velocity disturbances. Effects of Prandtl number on the critical conditions are discussed. Further, we propose a flow map which predicts azimuthal wave numbers and oscillatory mode over a wide range of aspect ratio for Pr=0.

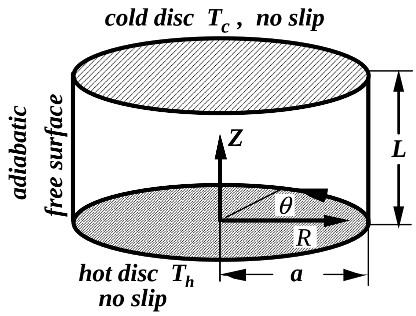

Fig.1 Schematics of liquid bridge

2. MODEL FORMULATIONS

A standard model of half-zone liquid bridge as shown in **Fig. 1** is adopted [20-22,30]. The liquid surface is assumed adiabatic, non-deformable and cylindrical. This shape is true under microgravity condition. There acts the Marangoni effect on the liquid surface. Fundamental equations are as follows.

$$\nabla \cdot \mathbf{U} = 0 \qquad (1)$$

$$\frac{\partial \mathbf{U}}{\partial \tau} + (\mathbf{U} \cdot \nabla)\mathbf{U} = -\nabla P + \nabla^2 \mathbf{U} \qquad (2)$$

$$\mathrm{Pr}\left(\frac{\partial \Theta}{\partial \tau} + (\mathbf{U} \cdot \nabla)\Theta\right) = \nabla^2 \Theta \qquad (3)$$

Initial conditions:

$$\mathbf{U} = 0, \quad \Theta = -0.5 \qquad \tau \le 0$$

Boundary conditions:
on both end plates (Z=0 and As):

$$\mathbf{U}_{(R,\theta,0)} = \mathbf{U}_{(R,\theta,As)} = 0,\ \Theta_{(R,\theta,0)} = +0.5,\ \Theta_{(R,\theta,As)} = -0.5$$

at the surface (R=1):

$$\partial \Theta / \partial R = 0, \quad \partial \mathbf{U}_Z / \partial R = -\mathrm{Re}\, \partial \Theta / \partial Z,$$

$$R^2 \partial (\mathbf{U}_\theta / R) / \partial R = -\mathrm{Re}\, \partial \Theta / \partial \theta, \quad \mathbf{U}_R = 0$$

These equations are equivalent to those reported last year. But slightly different definitions of non-dimensional variables are adopted as follows in order to enable simulations on Pr=0 fluid. The dimensionless parameters are the Prandtl number, the Reynolds and the Marangoni numbers defined as $Pr=\nu/\alpha$, $Re=\sigma_T \Delta T a/\mu\nu$ and $Ma=\sigma_T \Delta T a/\mu\alpha = RePr$, respectively. The non-dimensional variables are defined as; $\{R, Z\} = \{r/a, z/a\}$, $P = pa^2/(\nu\mu)$, $\boldsymbol{U} = \boldsymbol{u}a/\nu$, $\Theta = (T-T_m)/\Delta T$, $\tau = t\nu/a^2$, where $T_m = (T_h+T_c)/2$, $\alpha = \lambda/c_p \rho$, $\boldsymbol{u}$: velocity, p: pressure, c_p: heat capacity, ρ: density, λ: thermal conductivity, μ: viscosity and ν: kinematic viscosity.

3. NUMERICAL METHOD

These equations are discretized by a finite difference method with a modified central difference treatment for the convective terms [27] and non-uniform staggered grids. Non-uniform grids were adopted to increase the resolution. The radial velocities on the central axis were calculated by means of the method of Ozoe et al. [28]. The HSMAC scheme was used to proceed time evolution of velocity and pressure. For the sake of reducing computation time, the energy equation was solved by an implicit method. By this modification, computation speed was increased by a factor of 3 to 10. This method becomes more effective for smaller *Pr* cases. Time step $\delta\tau$ was chosen between $1x10^{-5}$ and $1x10^{-4}$. Also a fully implicit code was developed in this year. This code provides very fast calculation, however, only on the super computer. The critical Reynolds numbers were searched by means of fully explicit method with time step $\delta\tau$ between $1x10^{-6}$ and $5x10^{-6}$. In this work, we gave 3-D disturbances by imposing very small random value (average value=0, standard deviation of 10^{-8}) on velocities on every grid points, as embryos of disturbance. These numerical disturbances incubate 3-D disturbances automatically and they start growth with time. A two dimensional (2D) simulation code with the same scheme and 2D grids was run in order to obtain a 2D solution under the same conditions. If we adopt thermophysical properties of molten silicon, such as $\nu=2.5x10^{-7}$ [m^2/s], non-dimensional time span $\Delta\tau=1$ corresponds approximately to 100 seconds for a liquid bridge of 5.0mm in radius. The program was run on an MPU of the Fujitsu VPP700 at the Computer Center of Kyushu University or Compaq XP-1000. The validity of the numerical codes has been reported for *Pr*=1.02 fluid [4,5] and also for *Pr*=0.01 fluid [20,21,22] by comparing the first critical Reynolds numbers (Re_{c1}) with those of linear stability analyses[12,13], and also comparing the second critical Reynolds numbers (Re_{c2}) with available results [17,18]. By our codes, we determined both the first and the second critical Reynolds numbers within few percent of error from the reported values.

4. RESULTS

4.1 Results with *Pr*=0

4.1.1 Steady 3-D flow and Re_{c1}

As shown in the previous Annual Report [30] Marangoni flow is induced by a linear axial temperature distribution but the temperature field would never be disturbed by any change of flow pattern. Transient numerical simulations with a small value of *Re* (Re>Re_{c1}) shows an exponential growth of 3-D disturbance with time with a growth rate constant β. Mode of the 3-D Marangoni flow is characterized by the azimuthal wave number, *m*. Then, the growth process of disturbance would be expressed as:

$$X(\tau) = X(0)\sin(m\theta)\exp(\beta\tau)$$

Origin of the 3D flow in half-zone of *Pr*=0 fluid was explained as a shear instability caused in the return flow. Under very small *Re*, the cold return flow goes back along the axis as a coaxial plume. As increasing *Re*, the flow rate increases and large amount of returning liquid meets at the axis near the cold plate. Shear instability occurs at a certain flow rate and the return flow is deformed oblate and cold fluid flows back obliquely and a 3-D flow pattern is formed [27]. In a short bridge (*As*=0.6), 3-D disturbance is characterized as *m*=3. In bridges of *As*=0.8-1.4, disturbance with *m*=2 is incubated and increases its amplitude until its steady state. In longer bridges, *As*=1.6 - 2.0, the most dangerous disturbance is characterized as *m*=1. In liquid bridges of *Pr*=0 fluid, the 3-D steady flow pattern of *m*=2 is dominant over wide range of *As* (*As*=0.8-2.0) except for the 3-D steady flow with *m*=3 at *As*=0.6. However, it should be noted that the most dangerous mode for *As*=1.8 and 2.0 is *m*=1.

Thus determined Re_{c1} are smaller than those for finite Prandtl number fluids (*Pr*=0.01 and 0.02) obtained by linear stability theory [12,13] as shown in **Fig.2**. At *As*=1.6, the most dangerous mode shows a cross-over from 2 to 1. Correspondingly, the first critical Reynolds number, Re_{c1}, shows a local maximum at *As*=1.6.

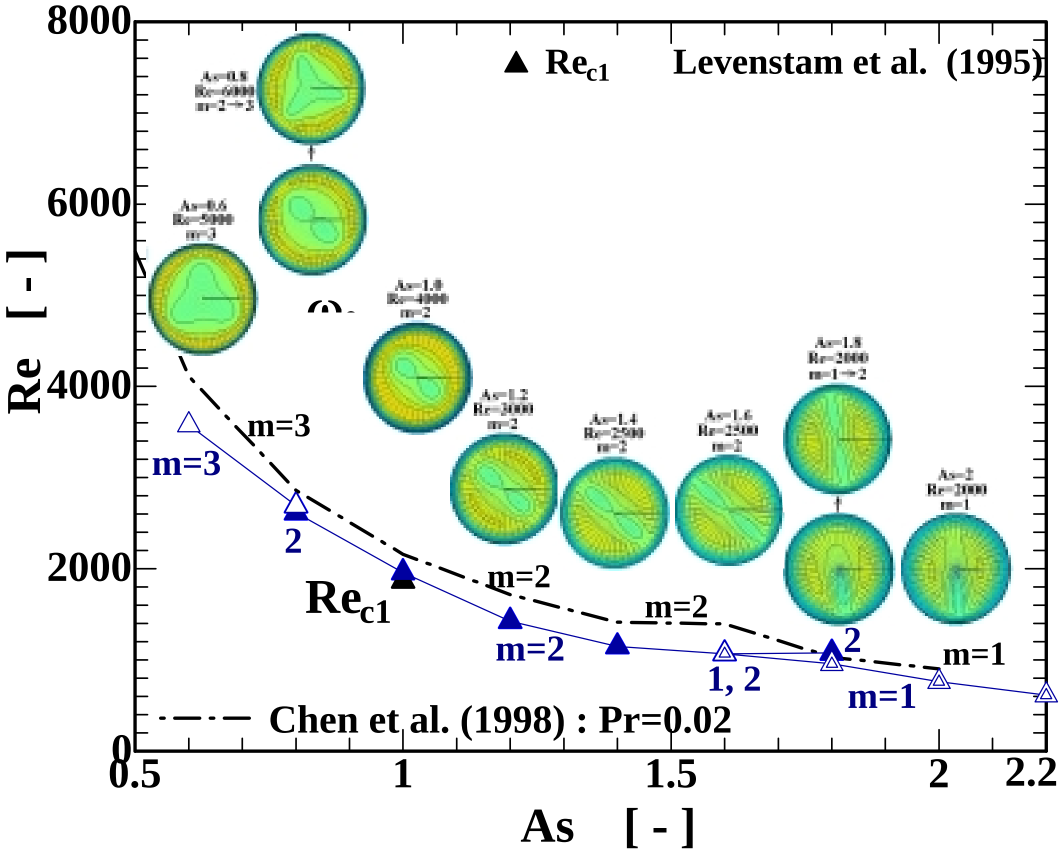

Fig.2 The first critical Reynolds number as a function of the aspect ratio.
Keys and thin solid lines: Present results for *Pr*=0 fluid.
Thick dotted line: First critical Reynolds numbers for *Pr*=0.02 by linear stability Analysis of Chen et al. [13]
Insets show distribution of *Z* component of volticity on a horizontal cut plane at *Z*=0.5*As*.

4.1.2 Oscillatory flow and the second critical Reynolds number, Re_{c2}

These steady 3-D flows become unstable against time-dependent 3-D disturbances and start oscillation at and beyond the second critical Reynolds number. In our previous report, we classified the 3-D oscillatory flow into 3 types and named them as follows; 1: (*m*+1) type: a time-dependent disturbance of *m*=1 is imposed on the basic steady flow of *m*, 2: (*m*-*T*) torsional-oscillation (twisting or back and forth action in azimuthal direction) of the longer axis of the cold plume with a wave number of *m*, and 3: (*m*-*R*) type; an oscillation mode accompanied by a rotating 3-D flow structure of *m*. The (*m*-*R*) type oscillatory flow is similar to the rotating 3-D flow in case of high *Pr* fluid. However, the 3-D flow in low *Pr* fluid is again hydrodynamic in nature.

Growth and decay rate constant, β_2, of the oscillation amplitudes depends on *Re* value. We can determine the second critical Reynolds number, Re_{c2}, from a plot of β_2 vs. Re, as reported in our previous works [20,21]. Thus determined values of Re_{c2} are plotted in **Fig.3** as a function of *As*. Oscillation mode depends on the aspect ratio; short bridge exhibits oscillatory flow of *m*=3. At around *As*=0.8, Re_{c2} shows a local maximum. In a range 0.9<As<1.2, (2+1) type oscillation is the most dangerous oscillation mode. Slightly longer liquid bridges, 1.3<As<1.6, a (2-T) type oscillation is the most dangerous mode. In longer liquid bridges, 1.8<As, a (2-T) type oscillation seems to be the most dangerous mode. However, the oscillation mode is complex and exhibits a long period alternation between (2-T) and (1-T) types.

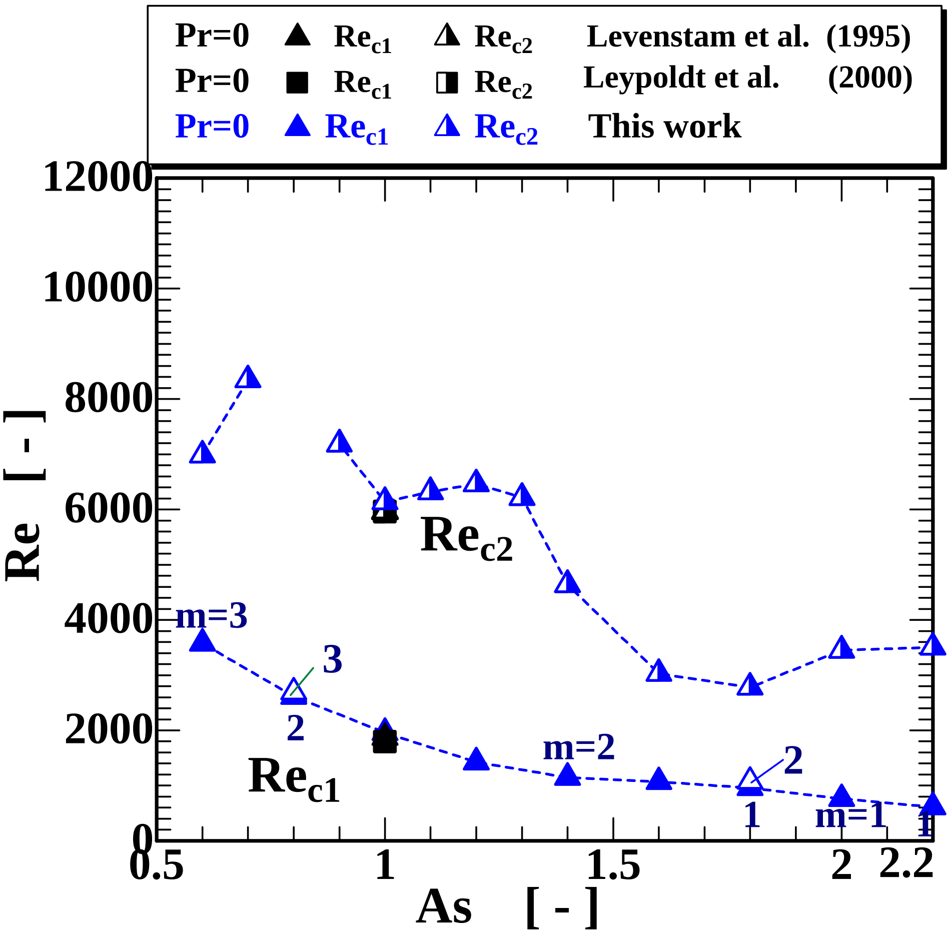

Fig.3 The second critical Reynolds number as a function of aspect ratio for *Pr*=0 fluid.

4.1.3 Flow map for *Pr*=0

Results of our simulations for *Pr*=0 are summarized in **Fig.4** as a flow map. This figure enables us a prediction of 3-D flow mode under a given condition, *As* and *Re*. It should be noted that *m* value changes with *Re* as well as *As*. At large value of *Re*, the oscillation becomes non-periodic and rather caotic and its power spectra show broadened multi frequency peaks as reported in our previous reports [20,21,22, 30]

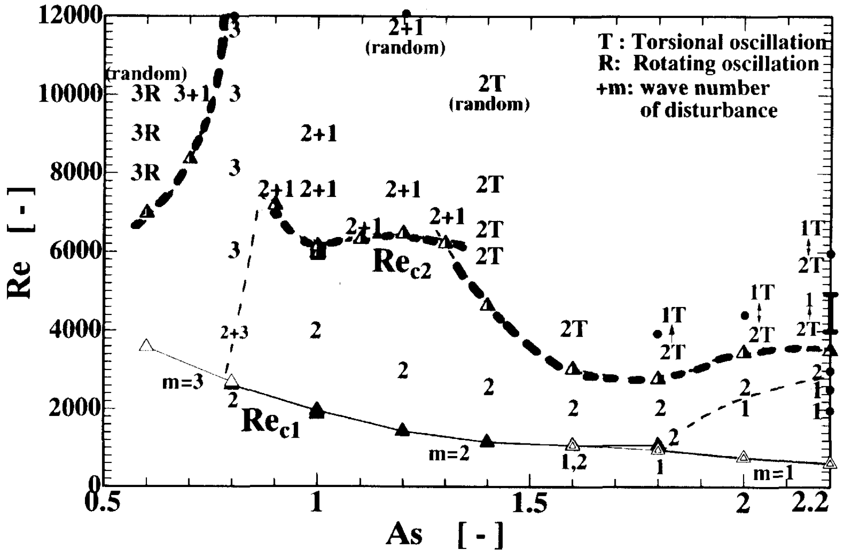

Fig.4 Flow map for Marangoni flow in half-zone liquid bridges of *Pr*=0 fluid.

Numeral beside each key represents *m* and the mode of oscillation.
Thin solid line: the first critical condition.
Thick dotted line : the second critical condition.
Thin dotted lines separate different flow structures among steady 3-D flows.

4.2 Results for *Pr*=0.01 and 0.02

4.2.1 First transition and Re_{c1}

For *Pr*=0.01 fluid, the critical conditions for the first flow transitions have been reported for *As*=1.0 and *As*=1.2 in previous papers [7,8,9] and *As*=1.4 and 1.8 [30]. With these small, but finite, values of *Pr*, a coupling between flow and temperature fields gives influences on stability limit of the axisymmetric steady Marangoni flow. Present results indicate that the coupling stabilizes the axisymmetric steady Marangoni flow, as shown in **Fig.5**.

4.2.2 Second flow transition and Re_{c2}

The second critical Reynolds numbers are plotted in **Fig. 6** together with the Re_{c1} and flow patterns. 3-D oscillatory flows in short liquid bridges are similar to that of *Pr*=0. At around *As*=1.2, Re_{c2} exhibits steep increase. Especially for *Pr*=0.02 fluid, a rotating type oscillatory flow is incubated at very large temperature differences. Under much larger *Re* values, rotating oscillatory flow tends to occur over wide range of *As*.

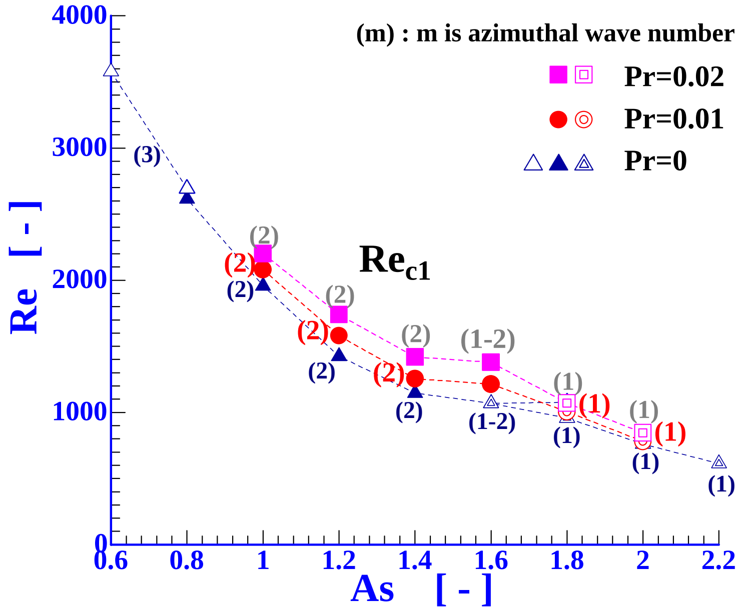

Fig.5 Effect of the Prandtl number on the first critical Reynolds numbers.

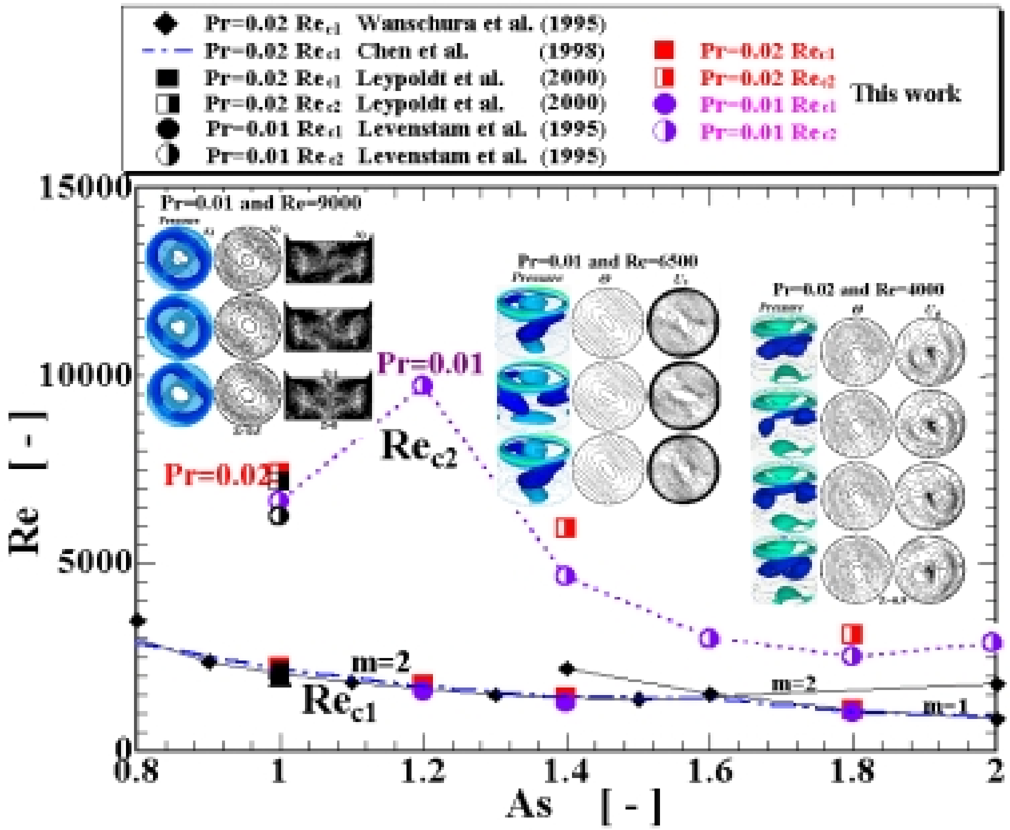

Fig.6 Summary of the critical conditions for low Pr fluid liquid bridges and typical types of oscillation under slightly super-critical conditions.
At Re=9000, Pr=0.01, As=1.0 : (2+1) type oscillation.
At Re=6500, Pr=0.01, As=1.4 : (2-T) type oscillation.
At Re=4000, Pr=0.02, As=1.8 : (1-T) type oscillation.

5. CONCLUSION

A set of 3-D numerical simulations was conducted to investigate the behavior of the Marangoni flow in half-zone liquid bridges of low Prandtl number fluids, including the limit of *Pr*=0, 0.01 and 0.02. The results elucidated the first and the second critical Reynolds numbers of the flow transitions of Marangoni flow as a function of the aspect ratio for the case of *Pr*=0 fluid. A flow map was proposed to predict flow patterns, oscillation mode for liquid bridges of *Pr*=0 fluid. For fluids of *Pr*=0.01 and 0.02, the first and the second critical conditions were determined.

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